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Effective potential evaluations in a modified PQCD

A. Cabo Montes de Oca[1,2,a], D. Martínez-Pedrera[1,2]

[1] International Center for Theoretical Physics, Strada Costiera 111, Trieste, Italy
[2] Group of Theoretical Physics, Instituto de Cibernética, Matemática y Física, Calle E, No. 309, Vedado, La Habana, Cuba

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Abstract. A procedure discussed in a previous work for properly defining the Feynman diagrams at any number of loops in a modified version of PQCD, is employed here to evaluate some zero- and one-loop corrections to the effective potential, as functions of the gluon and quark condensate parameters. The calculated terms indicate an instability of massless QCD under the development of quark condensates even in the absence of the gluon one. Therefore, a mechanism is suggested for the dynamical generation of quark masses and condensates. The absence of indications coming from lattice calculations to this possibility could be determined by the current limitations in treating fermion determinants.

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1 Introduction

The gauge invariance properties of the modified version of perturbative QCD previously considered in [2–8] have been discussed in [1]. In this work it was also possible to remove singularities in the Feynman diagrams produced by the presence of delta-function terms in the free propagators. Two basic ideas allowed for this outcome. One was the use of dimensional regularization to extend the appearing $\delta(0)$-like singularities to continuous D dimensions, in the way early introduced by Capper and Leibbrandt [9]. The second one was the use of Nakanishi infrared regularization for all the propagators. This procedure leads to the vanishing of all the remaining singularities, in the form of a Feynman propagator evaluated at zero momentum. In addition, the discussion in [1] suggested the identification of the propagators evaluated in [8], as modified tree Green functions. It can be noted that they also have zero order in the series expansion in the coupling g, after considering the independent parameters, the proper g, in addition to the gluon and quark condensate parameters after having been multiplied by g^2. This transformation seems to allow for a rearrangement of the loop expansion in a form for which the propagators derived in [8] could play the role of new tree Green functions. Here, we only consider one-loop terms in which a summation over the zero order in the coupling self-energy insertions is done.

Therefore, this work is devoted to an evaluation of selected one-loop contributions to the effective potential in order to get a sense of the possible physical predictions of the dynamical generation of quark and gluon condensates. In addition, a sample calculation of a particular two-loop term is done in order to get a measure of the influence of higher corrections. It should be said that the usual definition of the effective potential is considered in this work in order to avoid the property of being unbounded from below of the directly evaluated Cornwall–Jackiw–Tomboulis (CJT) potential [10, 11]. The approximation to be done consists in inserting all the condensate dependent parts of the one-loop self-energy corrections into the free propagators, by afterwards employing these dressed propagators in the usual zero- and one-loop corrections, as proposed in [1]. These corrected propagators precisely are the ones that generated the constituent masses in the Refs. [4, 8]. Since this re-summation includes higher order diagrams, a full discussion of the gauge parameter independence was not allowed in the present work. Therefore, we adopted the special gauge $\alpha = 1$ which greatly simplifies the evaluations. It can be added, however, that its special character is being identified in the current literature for the Feynman gauge. This is connected with the development of the pinch technique, in which the end gauge invariant results coincide with the ones in the Feynman gauge [12].

The resulting contribution to the effective potential indicates a generation of dynamical quark and gluon condensates. In the present approximation the evaluated potential shows the behavior of being unbounded from below. This instability becomes stronger by increasing the gluon condensate. However, even in the absence of the gluon condensate, the quark one is dynamically generated. This outcome is consistent with the fact that the finite temperature deconfinement transition should not drastically affect the masses of the heavy quarks.

[a] e-mail: cabo@cidet.icmf.inf.cu

The property of being unbounded from below and the dependence of the quark condensate signals the need for higher approximations in defining a minimum for the potential. It can be noted that the arising picture precisely resembles the one advanced in [8], as a possible consequence of the close analogies between the initial wave-function for the Wick expansion and the BCS states in superconductivity theory [3, 8]. At low values of the quark condensate parameter, the gluon one develops a minimum. It would be surprising that the value of mq required for the stabilization could be as high as $m_{\text{top}} = mq = 175$ GeV (as given by the pole of the quark propagator for high mq values [8]). However, we cannot yet disregard this possibility, and the search for an estimation of the stabilizing terms should be considered.

In addition, a particular two-loop contribution to the potential as a function of the quark condensate was also evaluated. The outcome for this quantity turned out to be finite after including the corresponding quark condensate dependent part of the usual quark counterterm. This result gave us confidence that the renormalization procedure can be well implemented in the modified theory. However, a careful discussion of this question should be considered in more detail.

The question of the gauge parameter dependence of the calculations was addressed in [1]. The gauge invariance of the physical quantities and the validity of the same Ward–Takahashi–Slavnov identities as in PQCD were argued there. This result corroborates the calculations supporting this validity in [4, 6, 7]. It is clear that the modified theory is a multi-parameter one, in which the implementation of the gauge invariance should be valid in each order of a triple series expansion. However, the perturbative series is also associated to a massless theory. Therefore, as in the simpler case of scalar electrodynamics, special partial summations should be made in order to get an alternative diagram expansion incorporating the condensate constants defining the new scales. Moreover, the implementation of gauge parameter independence turns out to be an additional requirement for the mentioned re-summations. However, this problem is a subtle one and needs a separate study, to be performed elsewhere.

The work is organized as follows: in Sect. 2, the propagators employed in the calculations are presented and the effective action vacuum diagrams described. The zero- and one-loop potential contributions are discussed in Sect. 3. Section 4 is devoted to an exposition and a discussion of the results of the evaluations done. Appendix A describes the evaluation of the sample two-loop term of the effective action. In the summary the main results of the work are briefly reviewed.

2 Propagators and effective action

In the next sections the evaluation of the effective potential including zero- and one-loop corrections will be considered. The contributions will be calculated by inserting the infinite ladder of condensate dependent one-loop self-energy parts in the original free propagators following the rules defined [8]. The propagators for quarks and gluons, as well as for the condensate lines defined in [8] are given as

$$\begin{aligned}
G^{ab}_{g\mu\nu}(p,m) =& \frac{\delta^{ab}}{(p^2-m^2+\mathrm{i}\epsilon)}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2+\mathrm{i}\epsilon}\right) \\
&+\frac{\alpha p_\mu p_\nu}{(p^2+\mathrm{i}\epsilon)^2}\delta^{ab} \\
=& \frac{\delta^{ab}}{(p^2-m^2+\mathrm{i}\epsilon)}\left(g_{\mu\nu}-\frac{\beta p_\mu p_\nu m^2}{(p^2+\mathrm{i}\epsilon)^2}\right), \\
\beta =& 1,
\end{aligned} \tag{1}$$

$$G^{f_1 f_2}_q(p,M,S) = \frac{\delta^{f_1 f_2}}{\left(-p_\mu\gamma^\mu\left(1-\frac{M^2}{p^2}\right)-\frac{S_{f_1}}{p^2}\right)}, \tag{2}$$

$$\chi^{ab}(p) = -\frac{\delta^{ab}}{p^2+\mathrm{i}\epsilon}, \tag{3}$$

$$G^{ab}_m = -\frac{\mathrm{i}m^2}{g^2}\delta^{ab}\,\delta(p), \tag{4}$$

$$G_S = \frac{\mathrm{i}4\pi^4 S_f}{g^2 C_F}\delta^{ab}\delta^{f_1 f_2}\delta(p), \tag{5}$$

where (1)–(3) are the gluon, quark and ghost propagators respectively and (4) and (5) the gluon and quark condensate ones. In this work we will adopt the general conventions for the spinor, color and Lorentz groups, free propagators and interaction vertices of [13].

The parameters m^2, M, S_f are related with the constants C_g and C_q [1, 8] characterizing the gluon and quark condensates, as follows:

$$-m^2 = m_g^2 = \frac{6g^2 Cg}{(2\pi)^4}, \tag{6}$$

$$S_f = \frac{g^2 C_F}{4\pi^4}C_q, \tag{7}$$

$$m^2 = fM^2, \quad f = \pm\left(\frac{3}{2}\right)^2, \tag{8}$$

$$\begin{aligned} g^2 &= g_0^2\mu^{2-\frac{D}{2}}, \\ D &= 4-2\epsilon. \end{aligned} \tag{9}$$

In these relations the parameter $f = \pm(\frac{3}{2})^2$ will be considered for both values of its sign. However, it should be underlined that only the negative value was implied by the construction done in [3]. In that work, it followed that if the parameter C_g is a positive one, then the gluon mass in the lowest approximation becomes tachyonic ($m^2 = -\frac{6g^2 C_g}{(2\pi)^4}$). On the other hand, the constituent mass for light quarks M becomes real. However, as it is simple to evaluate the expressions for the positive choice of f, sometimes it will be done below. It should be mentioned that for this value of the parameter, the gluon mass is real. Moreover, its value is near the estimated one due to other studies [14], $m = 0.5$ GeV. In contrast, the constituent mass gets a tachyonic value. However, in our view, a study of the physical justification of the positive choice of f deserves to be done, since the evaluations in this case could be related with the results of the approach in [15].

It should be explicitly noted that only one flavor is assumed to be condensed in the present discussion. This condition was chosen because at the present level of approximation, the consideration of various flavors will simply lead to the addition of identical fermions contributions to the potential. The question of the possible interference of various quark condensates, since it needs higher order approximations for its appearance, will be relegated to further studies. It is clear that this is a relevant point, because only in the case that the presence of various kinds of such condensates will be rejected by the system, the dynamical generation of only one (main) quark condensate will be preferred. This could occur, for example, thanks to the presence of terms in the effective potential growing in value when more than one condensate is present. This effect seems to be possible, but its consideration needs at least three-loop corrections, in which different quark loops can start to appear [8].

The collection of zero- and one-loop diagrams which were evaluated are illustrated in Fig. 1. The diagram Γ_1 is the only one not having closed loops, that is, a tree correction. Γ_2 and Γ_3 show the usual gluon and quark one-loop corrections associated to the propagators (1) and (2) respectively. Further, the diagrams Γ_4, Γ_5, Γ_6 and Γ_7 are related with the one-loop corrections being "descendant" from the two-loop ones due to the cancellation of one of the two-loop integrals by a condensate propagator, and with the insertion of all self-energy insertions independent of the coupling g leading to the propagators (1) and (2) in the other two lines [1, 8].

Finally in Fig. 2 the diagrams Γ_8 and Γ_9, Γ_{10} and Γ_{11} are defined as follows: Γ_8 is the standard diagram for the two-loop correction including all the coupling independent self-energy insertions in its internal lines; Γ_9 is the same contribution as Γ_8 but taken in the limit $S \to 0$, which is substracted in order to consider only the quark condensate dependent part of the two-loop term. The limit $S \to 0$ is indicated by the rings in the fermion lines. Finally Γ_{10} is the g^2 contribution associated to the fermion counterterm and Γ_{11} is the same contribution in the limit $S \to 0$ and is substracted, again, in order to only consider the quark condensate dependent part of the potential. The momentum integrations, in writing the diagram expressions, will be taken in Minkowski space, for afterwards we perform the Wick rotation. However, it should be made precise that in order to make the rotation without encountering poles, the sign of m^2 should be positive ($f = (\frac{3}{2})^2$). However, we will perform the rotation for the two signs of f without including the terms that could be incorporated by rounding the poles in the p_0 variables when deforming the integration contour if f takes its negative value. Therefore the results obtained for f negative should be interpreted as the evaluation of the effective potential in Euclidean field theory. That is, the evaluated quantity corresponds to the thermodynamical effective potential in the limit of zero temperature.

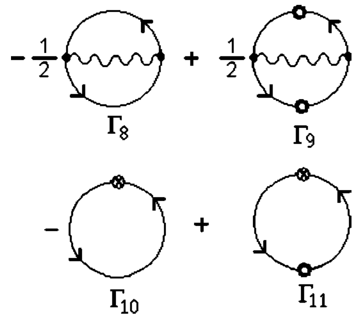

Fig. 2. The figure illustrates: **a** The two-loop fermion contribution Γ_8, considered in the work for getting a sense of the influence of the higher loops; **b** the subtracted diagram Γ_9, which is the same Γ_8 evaluated at $S = 0$; **c** the diagram associated to the fermion counterterm Γ_{10} and **d** this last diagram taken at $S = 0$ and indicated as Γ_{11}

The employed expression for the fermion renormalization constant is given by [13]

$$(Z_2 - 1) = -\frac{g^2 C_F}{(4\pi)^2}\frac{\beta}{\epsilon}\,,$$

$$T_{\mathrm{R}} = \frac{1}{2}, \quad C_G = N, \quad C_F = \frac{N^2 - 1}{N}\,.$$

As was remarked before, only one quark flavor will be considered for the present qualitative discussion, since up to this level all the quark flavors will produce a sum of identical contributions, having the same functional dependence on their respective condensates. However, the fact that light quarks exist furnishes a guiding principle in the sense that the mass acquired by them, if their quark condensates do not develop, should coincide with the parameter M [8] at the minimum of the effective potential.

We underline again that the value of the renormalized gauge parameter α is chosen to be equal to 1. This selec-

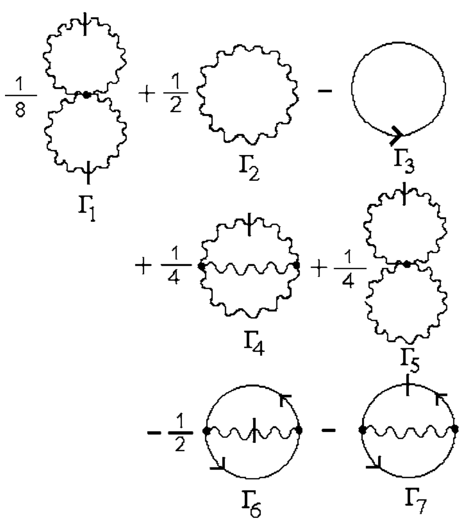

Fig. 1. The figure shows the seven Feynman diagrams defining the zero- and one-loop contributions evaluated in the work. The lines having cuts correspond to the condensate propagators. Therefore, although the associated diagrams may look as two loop ones, the delta functions associated to them effectively cancel one of the loop integrals

tion was done in order to simplify the evaluations, thanks to the simpler tensor structure of the gluon propagator in this gauge. In addition, since a study of the best approach for the re-summations of the Feynman diagrams has not yet been done, the prescriptions for the gauge invariance of the concrete calculations also have not yet been defined. The real possibility of solving these practical difficulties is however, indicated by the positive results on the gauge invariance properties obtained in [1].

3 Zero- and one-loop terms

The results for the evaluation of contribution to the one-loop effective potential Γ_1 to Γ_7 will be exposed below in consecutive order.

3.1 Zero-loop term

The direct substitution of the gluon condensate propagator (4) in the analytic expression associated to Γ_1, after evaluating all the Lorentz, spinor and color traces, leads to

$$\Gamma^{(0)} = -\frac{2m^4}{g^2} = -V^{(0)}.$$

That is, we have a positive potential proportional to m^4. As the one-loop term has zero order in the coupling g, in the expansion in powers of the parameters m and S_f, this term shows a power -2 of g, since the original diagram was of order two and there are two condensate lines in the diagram which reduce the power by four according to [1].

3.2 Standard one-loop terms

The sum of the one-loop terms corresponding to Γ_2 and Γ_3 in Fig. 1 have the form

$$\begin{aligned}\Gamma_{gf}^{(1)} =& \Gamma_g^{(1)} + \Gamma_f^{(1)} + \Gamma_S^{(1)} = -V_g^{(1)} - V_f^{(1)} - V_S^{(1)} \\ =& -\frac{\mathrm{i}}{2}\mathrm{Tr}\left[\log\left[G_g^{-1}(0)G_g^{-1}(m)\right]\right] \\ &+\mathrm{iTr}\left[\log\left[G_q^{-1}(0,0)G_q^{-1}(M,0)\right]\right] \\ &+\mathrm{iTr}\left[\log\left[G_q^{-1}(0,0)G_q^{-1}(M,S)\right]\right] \\ &-\mathrm{iTr}\left[\log\left[G_q^{-1}(0,0)G_q^{-1}(M,0)\right]\right]\ ,\end{aligned}$$

where the traces are in the momentum space, Lorentz and internal degrees of freedom as appropriate for each propagator. The momentum argument has been omitted in the Green functions defined in (1)–(5). These contributions have been expressed as the sum of a S independent term corresponding to the same diagrams evaluated at $S=0$, plus a S dependent contribution vanishing in the limit $S\to 0$. After calculating the Lorentz, spinor and color traces for the $S=0$ gluon and quark loops $\Gamma_g^{(1)}$ and $\Gamma_f^{(1)}$, and dimensionally regularizing the integral, it follows that

$$\Gamma_g^{(1)} = -\frac{(N^2-1)(D-1)}{2}\int\frac{\mathrm{d}p^D}{(2\pi)^D\mathrm{i}}\log\left[\frac{p^2}{p^2-m^2}\right]\ ,$$

$$\Gamma_f^{(1)} = 4N\int\frac{\mathrm{d}p^D}{(2\pi)^D\mathrm{i}}\log\left[\frac{p^2}{p^2-M^2}\right]\ .$$

But in both cases taking the derivative of the expressions over the parameters in the gluon and quark cases leads to simpler expressions. Then, after also performing the Wick rotation in the temporal momentum component according to

$$p_0 \to \mathrm{i}p_4\ ,$$

the derivative over the parameters expressions can be integrated in momentum space by employing the formula [13]

$$\int_{\mathrm{E}}\frac{\mathrm{d}p^D}{(2\pi)^D}\left[\frac{1}{p^2+L^2}\right] = \frac{B\left(\frac{D}{2},1-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})}(L^2)^{D-2},$$

in which L can be selected as m or M as appropriate for the gluon or quark terms respectively. The results of the integrals, after being integrated again but over the parameters, from their zero values to the original ones, and after also considering that the extended dimension D is such that the real part of $D-2$ is positive, allows one to write

$$\Gamma_g^{(1)}(m) = \frac{(D-1)(N^2-1)B\left(\frac{D}{2},1-\frac{D}{2}\right)(m^2)^{\frac{D}{2}}}{D(4\pi)^{\frac{D}{2}}\Gamma\left(\frac{D}{2}\right)}\ , \tag{10}$$

$$\Gamma_f^{(1)}(M) = -\frac{8NB\left(\frac{D}{2},1-\frac{D}{2}\right)(M^2)^{\frac{D}{2}}}{D(4\pi)^{\frac{D}{2}}\Gamma\left(\frac{D}{2}\right)}\ . \tag{11}$$

At this point, after considering the relations (6), (8) and (9) defining m and M as functions of the dimension D, and substracting the pole part in ϵ of (10) and (11), the minimal substraction result for the one-loop effective action is

$$V_g^{(1)}(m) = -\frac{(N^2-1)}{128\pi^2}m^4\left(-6\log\left(\frac{m^2}{4\pi\mu^2}\right)-6\gamma+5\right)\ , \tag{12}$$

$$V_f^{(1)}(M) = \frac{3(N^2-1)}{128\pi^2}M^4\left(-2\log\left(\frac{M^2}{4\pi\mu^2}\right)-2\gamma+3\right)\ . \tag{13}$$

The S dependent correction $\Gamma_S^{(1)}$, after all the trace evaluations are done, can be written as

$$\begin{aligned}\Gamma_S^{(1)} =& +\mathrm{iTr}\left[\log\left[G_q^{-1}(0,0)G_q^{-1}(M,S)\right]\right] \\ &-\mathrm{iTr}\left[\log\left[G_q^{-1}(0,0)G_q^{-1}(M,0)\right]\right], \\ =& 2N\int\frac{\mathrm{d}p^D}{(2\pi)^D\mathrm{i}}\log\left[\frac{p^2(p^2-M^2)^2}{p^2(p^2-M^2)^2-S^2}\right]\ .\end{aligned}$$

This integral, after the Wick rotation, is convergent in the limit $D\to 4$ and takes the form

$$\begin{aligned}\Gamma_S^{(1)} =& -V_{q,S}^{(1)} = -2N\int_{\mathrm{E}}\frac{\mathrm{d}p^D}{(2\pi)^D}\log\left[\frac{p^2(p^2+M^2)^2}{p^2(p^2+M^2)^2+S^2}\right] \\ =& -4N\int_0^\infty\frac{\mathrm{d}pp^3}{(4\pi)^2}\log\left[\frac{p^2(p^2+M^2)^2}{p^2(p^2+M^2)^2+S^2}\right]\ .\end{aligned}$$

3.3 One-loop terms descending from the two-loop gluon diagrams

After writing the analytical expressions for the diagram Γ_4 and evaluating the Lorentz, spinor and color traces, the expression can be rewritten in the form

$$\begin{aligned}\Gamma_{2g}^{(1,1)} &= -V_{2g}^{(1,1)} = \frac{(N^2-1)Nm^2}{4(2\pi)^D} \\ &\times \int \frac{\mathrm{d}p^D}{(2\pi)^D \mathrm{i}} \frac{(-6p^2D+2(D+11)\beta m^2-8m^4/p^2)}{(p^2-m^2)^2} \\ &= \frac{(N^2-1)Nm^2}{4(2\pi)^D} \\ &\times \int_{\mathrm{E}} \frac{\mathrm{d}p^D}{(2\pi)^D} \frac{(6p^2D+2(D+11)\beta m^2+8m^4/p^2)}{(p^2+m^2)^2}\,.\end{aligned}$$

In the second line of this equation the Wick rotation has been made. The integrals can be explicitly performed to give

$$\begin{aligned}\Gamma_{2g}^{(1,1)} &= -V_{2g}^{(1,1)} \\ &= \frac{(N-1)N\Gamma\left(1-\frac{D}{2}\right)\left[3D^2+2\beta(D+11)\left(1-\frac{D}{2}\right)\right]}{4(2\pi)^D(4\pi)^{\frac{D}{2}}} \\ &\times (m^2)^{\frac{D}{2}} \\ &+ \frac{2(N-1)N\Gamma\left(\frac{D}{2}-1\right)\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D(4\pi)^{\frac{D}{2}}\Gamma\left(\frac{D}{2}\right)}(m^2)^{\frac{D}{2}}\,.\end{aligned}$$

After applying the same procedure for the analytical expressions associated to the diagram Γ_5, the result is

$$\begin{aligned}\Gamma_{2g}^{(1,2)} &= -V_{2g}^{(1,2)} \\ &= \frac{(N^2-1)N(D-1)m^2}{2(2\pi)^D} \\ &\times \int \frac{\mathrm{d}p^D}{(2\pi)^D \mathrm{i}} \frac{1}{(p^2-m^2)}\left[1-\frac{\beta m^2}{p^2}\right] \\ &= -\frac{(N^2-1)N(D-1)m^2}{4(2\pi)^D} \\ &\times \int_{\mathrm{E}} \frac{\mathrm{d}p^D}{(2\pi)^D} \frac{1}{(p^2+m^2)}\left[1+\frac{\beta m^2}{p^2}\right] \\ &= -\frac{(N-1)N(D-1)(D-\beta)\Gamma\left(1-\frac{D}{2}\right)}{2(2\pi)^D(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}}\,.\end{aligned}$$

It is an interesting outcome that after adding these two contributions and removing the dimensional regularization limit, the result remains finite, a fact that also eliminates the logarithmic terms in the result. The total contribution of these terms for the potential at the end takes the form

$$\lim_{D\to 4}\left(V_{2g}^{(1,1)}+V_{2g}^{(1,2)}\right) = \frac{3f^2M^4}{8\pi^2}\,.$$

3.4 One-loop terms descending from the two-loop quark diagram

The last one-loop diagrams Γ_6 and Γ_7 correspond to the descendants of the two-loop terms having a closed fermion line. The integral expression obtained for them, after performing the Lorentz, spinor and color traces, are not so simple and we just numerically evaluated them in this work. The resulting integral expressions are

$$\begin{aligned}\Gamma_{2q}^{(1,1)} &= -V_{2q}^{(1,1)} \\ &= \frac{(N^2-1)m^2}{3(4\pi)^2}\int_0^\infty \mathrm{d}p p^3 \frac{(2p^2(p^2+M^2)^2+DS^2)}{(p^2(p^2+M^2)^2+S^2)^2}\,, \\ \Gamma_{2q}^{(1,2)} &= -V_{2q}^{(1,2)} \\ &= NS^2\int_{\mathrm{E}} \frac{\mathrm{d}p}{(2\pi)^4} \frac{(Dp^2+m^2-\mathrm{i}\epsilon)}{(p^2(p^2+M^2)^2+S^2)(p^2+m^2-\mathrm{i}\epsilon)}\,.\end{aligned} \tag{14}$$

In ending this section it can be noticed that all the "descendant" diagrams became finite ones.

4 Discussion

In this section, the results for the evaluation of the effective potential as a function of the condensate parameters M, S and the couplings constant g are presented. The calculations are mainly done for the negative sign of the parameter f, which defines the relation between the constituent quark and gluon mass parameters m and M through

$$m^2 = -fM^2\,.$$

As it was remarked before, only this selection was arising in [3, 4], because the constituent mass value evaluated there was satisfying the above relation with the negative sign. However, evaluation for positive f is also commented on sometimes below. It can be noted that for positive f the results for the potential are real, a fact that is not occurring for the more relevant case $f = -(\frac{3}{2})^2$. In addition, in this situation, it turns out that the gluon mass value m, once the gluon condensate $\langle g^2G^2\rangle$ is fixed, is $m = 0.5$ GeV, coinciding with the result obtained in [14]. However, for this positive sign of f, the absolute value of the constituent mass for light quarks will be also 333 MeV, but will be of tachyonic character. These results are perhaps corresponding with the alternative construction discussed in [15], a question that surely deserves examination. In any case, neither gluons nor quarks appear in Nature and perhaps both will be absent as real excitations in both descriptions, none of them showing asymptotic states after including more corrections [8].

Let us define, for the purpose of graphical illustration, the quantities $V(mq, M, g, \mu)$ and its imaginary part $V_{\mathrm{im}}(mq, M, g, \mu)$ (where mq is defined as $mq = MX$), as the sum of all the contributions to the effective potential (the negatives of the effective action terms) evaluated in the previous sections, after having been divided by the

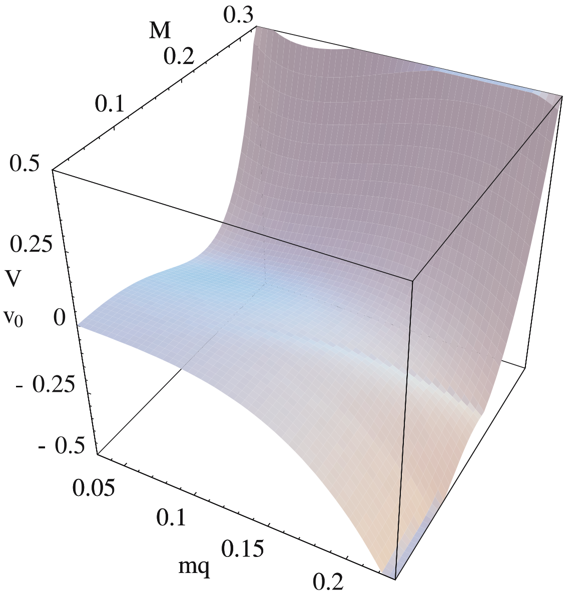

Fig. 3. The landscape picturing the dependence of the function V (the effective potential divided by $1/(8\pi^4)$) on the quark condensate (measured by mq) and the gluon one (measured by the constituent mass M). The values of the other parameters are $g = 2.74$ and $\mu = 6.8$ GeV. At $mq = 0$ the potential develops a minimum at a certain value of M which can be varied by changing the scale parameter μ. The dependence on mq indicates an instability upon the generation of values of mq which is not controlled at large mq values. It can be also seen that even at zero value of the gluon condensate ($M = 0$) the potential remains unbounded from below. That is, the sole presence of the quark condensate also makes the system unstable under the generation of mq from the state at $mq = 0$. This property suggests that the instability effect is not destroyed by the deconfinement transition at high temperatures, as it should be expected

constant factor $1/(8\pi^4)$. In the various figures below, the dependence of the quantity V, or its imaginary part, V_{im}, is plotted as a function of two of its arguments selecting the others as given by characteristic values of interest in the present state of the discussion. The plots are associated to the relevant case $f = -(3/2)^2$ and sometimes comments on the effect on the graphs of changing the sign of f will be given.

Figure 3 illustrates the behavior of the effective potential as a function mq and the constituent mass M. Both quantities are defined in the text in terms of the quark condensate and the gluon one through

$$X = \frac{mq}{M} = \frac{S^{\frac{1}{3}}}{M} = \frac{1}{M}\left(\frac{gC_F}{4\pi^4}C_f\right)^{\frac{1}{3}},$$

$$M^2 = \frac{m^2}{f} \quad \text{for } M \text{ real}.$$

The value of the coupling g selected for the plot was $g = 2.74$, which corresponds to a strong coupling value $\alpha = \frac{g^2}{4\pi}$ being near 0.6. In addition the mass scale parameter value $\mu = 6.8$ GeV was fixed. Note that the minimum at zero quark condensate $mq = 0$ is lying near 200 MeV, which is lower but close to the constituent mass value $M = 333$ MeV. It is interesting that, in order to fix the minimum of the potential for $mq = 0$ at this value of M, one requires a relatively large value of μ. As it can be observed, the landscape of the potential makes clear that the system at $mq = 0$ dynamically develops a gluon condensate parameter with a potential similar in form to the Savvidy one in the early chromomagnetic field models [16, 17]. It can be seen that the system at zero values of the parameters shows an instability upon the generation of both gluon and quark condensates. The effect is higher for the dynamical generation of the quark one. It can also be observed that the increasing of the gluon condensate parameter leads to an increment of the instability of the generation of the quark condensate. These properties are supporting the expectation expressed in [4, 8] about the color coupling being able to produce a sort of superconductivity effect capable to generate intensive quark condensates, resembling the Ginzburg–Landau fields. If such an effect is really occurring in Nature, the top condensate model could emerge as a possible effective field theory determined by the strong interactions of QCD. In this case, the Higgs field may be nothing other than the top condensate value as proposed in [18]. This occurrence could also explain the similarities between the properties of the quark mass spectrum and the spectrum of superconductivity systems, underlined in the "democratic symmetry breaking" analysis in [19].

Figure 3 clearly shows that, in the framework of the present approximation, and for reasonable values of the coupling ($\alpha = 0.6$), there are no terms that control the instability for the generation of the quark condensate. Therefore, it becomes clear that the determination of a minimum for the potential should come from higher order contributions. In [8], this behavior was guessed to occur thanks to the color interaction between quarks. Therefore, under the assumption that the technique used in this work is well describing massless QCD, it follows that this theory could dynamically develop quark condensates and masses. This outcome could be another realization of the dimensional transmutation effect [20]. A requirement for the next corrections to produce helpful results for phenomenology is that the stabilizing potential at large mq values behaves in such a way that its dependence on M assures that the extreme point occurs at low values of M. Then, the constituent mass could be fixed to the observable value near 333 MeV, by selecting appropriate values for the coupling and the scale parameter. Also the value of $mq \sim 175$ GeV should be allowed to be fixed.

Figure 4 shows the value of the imaginary part V_{im} of the potential as a function of M and mq. Note that the dependence on mq is not rapidly growing, a behavior that, if maintained for large mq values and in higher approximations, will indicate an increasing stability of the vacuum being proved for the interesting region of high values of mq. The picture is for $f = -(\frac{3}{2})^2$. For a positive value of f the imaginary part of the potential vanishes. More generally, it can be remarked that all the other types of pictures shown in this section, after being plotted for a positive

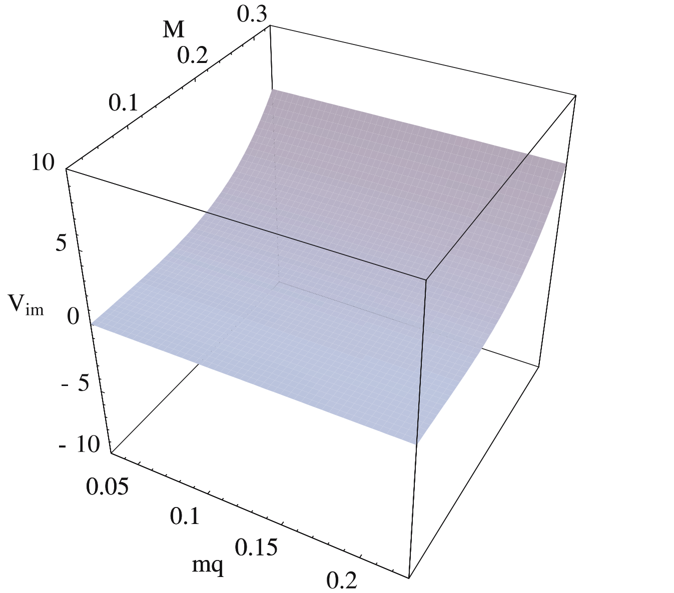

Fig. 4. The plot of the function $V_{\rm im}$ for the same range of mq and M used in Fig. 3 and the same fixed parameters $g = 2.74$, $\mu = 6.8$ GeV and $f = -(3/2)^2$. It can be seen that the ratio between the imaginary and the real part of the potential near the minimum at fixed mq decreases for the higher values of mq. Also, the imaginary part tends to be zero if the gluon condensate is disregarded in first approximation. For $f = (3/2)^2$ the potential is real for all the parameter values

value of f, show a very similar behavior. Further, Fig. 5 shows the dependence of the potential on the variable mq and the gauge coupling g. Here the mass parameter M was fixed to 333 MeV, and again μ is taken as 6.8 GeV, which fixes the minimum in the variable M at $mq = 0$ to be near 333 MeV. It should be recalled that we are considering that only one quark is being condensed. Therefore, the light quarks which in the present discussion do not develop their own condensates, should show at the considered level of approximation the observable value of light constituent

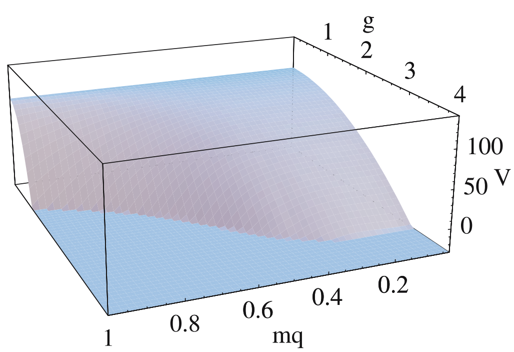

Fig. 5. The potential plotted as a function of mq and g for fixed values of $M = 333$ MeV and $\mu = 6.8$ GeV. It can be observed that for each value of mq, there is critical coupling g below which the potential becomes negative, that is, lower than its value at vanishing condensates. This critical coupling decreases when X grows

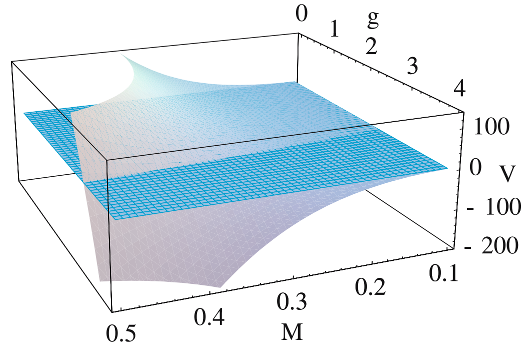

Fig. 6. The plot of V as a function of M and g fixing values of $mq = \mathrm{M}$ GeV and $\mu = 6.8$ GeV. It shows that there is a critical value of the strong coupling g below which the potential becomes positive for non-vanishing gluon condensates ($M > 0$)

masses. Since this quantity is fixed by the value of M, the graphics selected to be evaluated are always chosen to show a minimum near $M = 333$ MeV for $mq = 0$. This picture illustrates how the potential becomes negative (lower than its value at zero condensate state) when the coupling increases its strength over an amount fixed by the value of the quark condensate parameter mq. The greater the value of mq, the smaller becomes the critical coupling.

Figure 6 shows the dependence of the potential on the gluon condensate and the coupling constant for fixed values of $X = mq/M = 1$ and $\mu = 6.8$ GeV. It can be seen that below a certain critical coupling value near 2, for all values of M, the zero gluon condensate state is stable. Increasing the value of X is not destroying this property, and the value of the critical coupling is simply diminishing for larger X values.

5 Summary

The physical implications of a modified perturbation expansion for QCD are investigated. Employing the procedure for well defining the diagrams of the proposed expansion introduced in [1], zero- and one-loop contributions to the effective potential are evaluated. The potential, in the considered approximation, indicates an instability of massless QCD upon the generation of quark condensates. At the considered approximation, there are no terms making the potential bounded from below. Thus, the next corrections should determine a minimum. Thus, the results in the adopted approximations signal the dynamical generation of quark condensates and masses. However, improved evaluations in order to establish the obtained indications should be performed. It can be remarked that the source for this effect could not yet be detected by numerical studies, possibly because lattice QCD results are still limited in the consideration of the fermion determinants.

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Appendix : A two-loop sample term

Let us evaluate in this appendix a sample two-loop term for checking its influence on the zero- and one-loop results. We will consider the full dependence on the quark condensate associated to the diagram Γ_8 in Fig. 2 defined by a fermion loop formed with propagators (2), showing two quark–gluon interaction vertices. Therefore, the same diagram expression but evaluated at $S=0$ will be substracted from this contribution. This term is associated with Γ_9 in Fig. 2. This substraction simply corresponds to the same analytic expression of the diagram, but taken for $S=0$, and is represented by the same figure, but having small rings in the quark propagators. As the diagram associated to the fermion counterterm of the standard massless QCD Γ_{10} (of order g^2, and therefore needed for renormalization at one-loop level) is also depending on the condensate parameter S, the same kind of substraction is done of the $S=0$ counterterm associated to Γ_{11}, in which, again, the ring in the quark line means the evaluation in $S=0$.

The substracted terms exactly give the full two-loop term formed by two quark propagators and one gluon line of the theory in the absence of the fermion condensate. As mentioned before, we will postpone the evaluation of the full two-loop gluon parameter dependence to further studies. The main reason for doing so is that for these terms it is more relevant to precisely define the way in which the renormalization should be done within the considered scheme. As it is discussed in [1], at the two-loop level there will appear an additional quark condensate dependence coming from two-loop diagrams being descendant from the higher loop terms of the original expansion. From exploring evaluations already done, we know, however, that it seems possible to cancel the two-loop infinities by renormalizing the condensate parameters. However, a clearer understanding of the structure of the allowed counterterms in the modified expansion is yet desirable before evaluating the two-loop terms.

After calculating the spinor and color traces in the analytic expressions corresponding to the Feynman graphs appearing in Fig. 1, the considered contributions to the effective potential can be written in the form

$$\begin{aligned}
&\Gamma^{(2)}_{fg}(M,S)\\
&= -\frac{(N^2-1)g^2}{4}\int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}}\frac{\mathrm{d}^D q'}{(2\pi)^D \mathrm{i}}\frac{1}{((q-q')^2-m^2)}\\
&\quad\times\frac{1}{(q^2(q^2-M^2)^2-S^2)(q'^2(q'^2-M^2)^2-S^2)}\\
&\quad\times\Bigg\{-4q^2q'^2(q^2-M^2)(q'^2-M^2)\Bigg[(D-2)q\cdot q'-\beta\frac{m^2}{(q'-q)^2}\\
&\quad\times\left(q\cdot q'-\frac{2q\cdot(q'-q)q'\cdot(q'-q)}{(q'-q)^2}\right)\Bigg]\\
&\quad+4\left(D-\frac{\beta m^2 S^2}{(q'-q)^2}\right)q^2q'^2\Bigg\}, \qquad \text{(A.1)}
\end{aligned}$$

$$\begin{aligned}
&\Gamma^{(2)}_{fg}(M,0) = -\frac{(N^2-1)g^2}{4}\\
&\times\int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}}\frac{\mathrm{d}^D q'}{(2\pi)^D \mathrm{i}}\frac{(-4)}{((q-q')^2-m^2)(q^2-M^2)(q'^2-M^2)}\\
&\times\left[(D-2)q\cdot q'-\beta\frac{m^2}{(q'-q)^2}\left(q\cdot q'-\frac{2q\cdot(q'-q)q'\cdot(q'-q)}{(q'-q)^2}\right)\right], \qquad \text{(A.2)}
\end{aligned}$$

$$\Gamma^{(2)}_{f\mathrm{C}}(M,S) = 4N(Z_2-1)\int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}}\frac{(q^2)^2(q^2-M^2)}{q^2(q^2-M^2)^2-S^2}, \qquad \text{(A.3)}$$

$$\Gamma^{(2)}_{f\mathrm{C}}(M,0) = 4N(Z_2-1)\int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}}\frac{q^2}{(q^2-M^2)}\,. \qquad \text{(A.4)}$$

It can be noticed that the mass dimension of the parameter S is equal to 3, which is a relatively high value. Therefore, the terms of the expansion in powers of S for the denominator of the integrand associated to $\Gamma^{(2)}_{gf}$ will have three powers of the momentum convergence factors for each power of S appearing in the expansion. The same effect occurs in the fermion counterterm $\Gamma^{(2)}_{f\mathrm{C}}$.

Then, it follows that the quantity

$$\begin{aligned}
\Gamma_{fg}(m,M,S,\epsilon) =& \Gamma^{(2)}_{gf}(M,S,\epsilon)-\Gamma^{(2)}_{gf}(M,0,\epsilon)\\
&+\Gamma^{(2)}_{f\mathrm{C}}(M,S,\epsilon)-\Gamma^{(2)}_{f\mathrm{C}}(M,0,\epsilon)\,,
\end{aligned}$$

which contains, by construction, the whole dependence of the considered two-loop term on the fermion condensate parameter S, turns out to be finite in the limit $D\to 4$ ($\epsilon\to 0$). This result is simply expressing the fact that the renormalization constant Z_2 of the massless QCD (determined in the absence of any condensate) is also able to extract the infinities from the single fermion condensate dependent contribution under study. As noticed before, according to the above described procedure, the substracted terms in addition with the two-loop ones not considered exactly correspond to the two-loop plus counterterm contributions in the absence of the fermion condensate. These terms, including the ones descending form the higher loops, according to the discussion in [1], were not considered here.

The finite contribution Γ_{fg} before passing to Euclidean variables can be written as the sum of the following three terms:

$$\Gamma^{(2)}_{fg} = \Gamma^{(2,1)}_{fg}+\Gamma^{(2,2)}_{fg}+\Gamma^{(2,3)}_{fg} \qquad \text{(A.5)}$$

$$\begin{aligned}
\Gamma^{(2,1)}_{fg} =& \frac{(N^2-1)g^2}{2}\int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}} \qquad \text{(A.6)}\\
&\times\int\left(\frac{\mathrm{d}^D q'}{(2\pi)^D \mathrm{i}}\frac{4(D-2)q\cdot q'}{((q-q')^2-m^2)(q'^2-M^2)}-\frac{4}{(4\pi)^2}\frac{q^2}{\epsilon}\right)
\end{aligned}$$

$$
\begin{aligned}
&\times \frac{S^2}{(q^2(q^2-M^2)^2-S^2)(q^2-M^2)} \\
&+(N^2-1)g^2 \int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}} \frac{\mathrm{d}^D q'}{(2\pi)^D \mathrm{i}} \frac{2(D-2) q\cdot q' S^4}{(q^2(q^2-M^2)^2-S^2)} \\
&\times \frac{1}{\begin{pmatrix} (q'^2(q'^2-M^2)^2-S^2)(q^2-M^2) \\ \times (q'^2-M^2)((q-q')^2-m^2) \end{pmatrix}},
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_{fg}^{(2,2)} =& -\frac{\beta m^2 (N^2-1) g^2}{4} \qquad \text{(A.7)}\\
&\times \int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}} \frac{\mathrm{d}^D q'}{(2\pi)^D \mathrm{i}} \frac{4q^2 q'^2 (q^2-M^2)}{(q^2(q^2-M^2)^2-S^2)} \\
&\times \frac{(q'^2-M^2)(2q^2q'^2 - q\cdot q'(q^2+q'^2))}{(q'^2(q'^2-M^2)^2-S^2)((q-q')^2-m^2)((q'-q)^2)^2},
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_{fg}^{(2,3)} =& \frac{\beta m^2 (N^2-1) g^2}{4} \qquad \text{(A.8)}\\
&\times \int \frac{\mathrm{d}^D q}{(2\pi)^D \mathrm{i}} \frac{\mathrm{d}^D q'}{(2\pi)^D \mathrm{i}} \frac{4q^2 q'^2}{(q^2(q^2-M^2)^2-S^2)} \\
&\times \frac{S^2}{(q'^2(q'^2-M^2)^2-S^2)((q-q')^2-m^2)(q'-q)^2},
\end{aligned}
$$

where the term showing the $\frac{1}{\epsilon}$ factor is associated to the fermion counterterm. It is responsible for the substraction of the divergent part of the remaining expressions.

After performing the Wick rotation, it is possible to eliminate the pole term in ϵ by using the identity

$$
\frac{1}{\epsilon} = \frac{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2}) \left[-q^2\right]^{2-\frac{D}{2}}}{\epsilon B(\frac{D}{2}, 2-\frac{D}{2}) B(\frac{D}{2}-1, \frac{D}{2}-1)} \int \frac{\mathrm{d}^D q'}{(2\pi)^D \mathrm{i}} \frac{1}{(q-q')^2 q'^2}.
$$

Then, the finite fermion condensate dependent contribution to the particular two-loop term evaluated here, in the limit $\epsilon \to 0$, can be expressed as follows:

$$
\begin{aligned}
V_{fg} &= -\Gamma_{ffg} = -v_0 \left[v_f^{(1)} + v_f^{(2)} + v_f^{(3)} + v_f^{(4)} + v_f^{(5)} \right], \\
v_0 &= \frac{4}{(4\pi)^4} (N^2-1) g^2 M^4 .
\end{aligned}
$$

The quantities $v_f^{(i)}, i = 1,2,3,4,5$ appearing above were reduced to simple 2D integrals after performing the angular integrations in the four-dimensional Euclidean space. They take the explicit forms

$$
\begin{aligned}
v_f^{(1)} =& -2X^6 \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}q' \\
&\times \frac{q^4 q'^4}{(q^2(q^2+1)^2+X^6)(q'^2(q'^2+1)^2+X^6)} \\
&\times \ln\left(\frac{q^2+q'^2+2qq'+f-\mathrm{i}\epsilon}{q^2+q'^2-2qq'+f-\mathrm{i}\epsilon} \right),
\end{aligned}
$$

$$
\begin{aligned}
v_f^{(2)} =& +X^6 \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}q' \frac{q^3 q'^3}{(q^2(q^2+1)^2+X^6)(q^2+1)} \\
&\times \left\{ \frac{1}{q'^2(q'^2+1)} + \frac{q^2+q'^2+f-\mathrm{i}\epsilon}{4qq'(q'^2+1)} \right. \\
&\times \ln\left(\frac{q^2+q'^2+2qq'+f-\mathrm{i}\epsilon}{q^2+q'^2-2qq'+f-\mathrm{i}\epsilon} \right) \\
&\left. - \frac{q^2+q'^2-\mathrm{i}\epsilon}{4qq'^3} \ln\left(\frac{q^2+q'^2+2qq'-\mathrm{i}\epsilon}{q^2+q'^2-2qq'-\mathrm{i}\epsilon} \right) \right\},
\end{aligned}
$$

$$
\begin{aligned}
v_f^{(3)} =& -X^{12} \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}q' \\
&\times \frac{q^3 q'^3}{\begin{pmatrix} (q^2(q^2+1)^2+X^6)(q'^2(q'^2+1)^2+X^6) \\ \times (q^2+1)(q'^2+1) \end{pmatrix}} \\
&\times \left\{ -1 + \frac{q^2+q'^2-\mathrm{i}\epsilon}{4qq'} \ln\left(\frac{q^2+q'^2+2qq'-f^2-\mathrm{i}\epsilon}{q^2+q'^2-2qq'-f^2-\mathrm{i}\epsilon} \right) \right\},
\end{aligned}
$$

$$
\begin{aligned}
v_f^{(4)} =& -\beta \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}q' q^3 q'^3 (q^2+1)(q'^2+1) \\
&\times \left\{ \frac{q^2 q'^2}{(q^2(q^2+1)^2+X^6)(q'^2(q'^2+1)^2+X^6)} \right. \\
&\left. - \frac{1}{(q^2+1)^2(q'^2+1)^2} \right\} \\
&\times \left\{ 1 + \frac{(q^2+q'^2 - \frac{(q^2-q'^2)^2}{f})}{4qq'} \right. \\
&\left. \times \ln\left(\frac{(q-q')^2+f-\mathrm{i}\epsilon}{(q+q')^2+f-\mathrm{i}\epsilon} \frac{(q+q')^2-\mathrm{i}\epsilon}{(q-q')^2-\mathrm{i}\epsilon} \right) \right\},
\end{aligned}
$$

$$
\begin{aligned}
v_f^{(5)} =& -\frac{\beta X^6}{2} \int_0^\infty \mathrm{d}q \\
&\times \int_0^\infty \mathrm{d}q' \frac{q^4 q'^4}{(q^2(q^2+1)^2+X^6)(q'^2(q'^2+1)^2+X^6)} \\
&\times \ln\left(\frac{(q-q')^2+f-\mathrm{i}\epsilon}{(q+q')^2+f-\mathrm{i}\epsilon} \frac{(q+q')^2-\mathrm{i}\epsilon}{(q-q')^2-\mathrm{i}\epsilon} \right),
\end{aligned}
$$

in which, as before, the dimensionless quantities X are given as follows:

$$
S = M^3 X^3 .
$$

The ϵ parameter is retained here, since it helps to regularize the integrals even in the Euclidean case when f is negative.

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